Finite Math - Fall 2018

Lecture Notes - 11/13/2018

Homework

- Section 4.7 - 21, 22, 23, 33, 35, 36, 37, 38
- Section 5.1 - 1, 2, 3, 4, 9, 11, 13, 17, 29, 30, 52, 54

Section 4.7 - Leontief Input-Output Analysis

Summarizing from Thursday's class, these are the steps to solving an input-output analysis problem:

(1) Find the technology matrix M and the final demand matrix D.

(2) Find $I - M$.

(3) Find $(I - M)^{-1}$.

(4) Find $X = (I - M)^{-1}D$.

(5) Interpret the answer in words.

Example 1. *An economy is based on two industrial sectors, coal and steel. In this economy, to produce* \$1 *worth of coal requires an input of* \$0.10 *from the coal sector and* \$0.20 *from the steel sector; and to produce* \$1 *worth of steel requires an input of* \$0.20 *from the coal sector and* \$0.40 *from the steel sector. The final demand (the demand from all other users of coal and steel) is* \$20 *billion for coal and* \$10 *billion for steel. Find the required output from each sector needed to produce this final demand.*

Solution. *First identify M and D:*

$$M = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \quad D = \begin{bmatrix} 20 \\ 10 \end{bmatrix}.$$

Then

$$I - M = \begin{bmatrix} 0.9 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

and

$$(I - M)^{-1} = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix}.$$

So, the required output is

$$X = (I - M)^{-1}D = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 28 \\ 26 \end{bmatrix}$$

So to meet the internal and final demands, \$28 *billion of coal and* \$26 *billion of steel must be produced.*

Example 2. *The economy of a small island nation is based on two sectors, agriculture and tourism. Production of a dollar's worth of agriculture requires an input of* \$0.20 *from agriculture and* \$0.15 *from tourism. Production of a dollar's worth of tourism requires an input of* \$0.40 *from agriculture and* \$0.30 *from tourism. Find the output from each sector that is needed to satisfy a final demand of* \$60 *million for agriculture and* \$80 *million for tourism.*

Solution. \$148 *million from agriculture and* \$146 *million from tourism.*

Section 5.1 - Linear Inequalities in Two Variables

Graphing Linear Inequalities in Two Variables. There are 4 types of linear inequalities

$$Ax + By \geq C \qquad Ax + By > C$$

$$Ax + By \leq C \qquad Ax + By < C$$

There is a simple procedure to graphing any of these. If equality is not allowed in an inequality, we call it a *strict inequality*, otherwise we simply call it an inequality.

Procedure.

(1) *Graph the line* $Ax + By = C$ *as a dashed line if the inequality is strict. Otherwise, graph it as a solid line.*

(2) *Choose a test point anywhere in the plane, as long as it is not on the line. (The origin,* $(0, 0)$ *is often an easy choice here, but if it is on the line,* $(1, 0)$ *or* $(0, 1)$ *are also easy points to check.)*

(3) *Plug the point from step* (2) *into the inequality. Is the inequality true? Shade in the side of the line with that point. If the inequality is false, shade in the other side.*

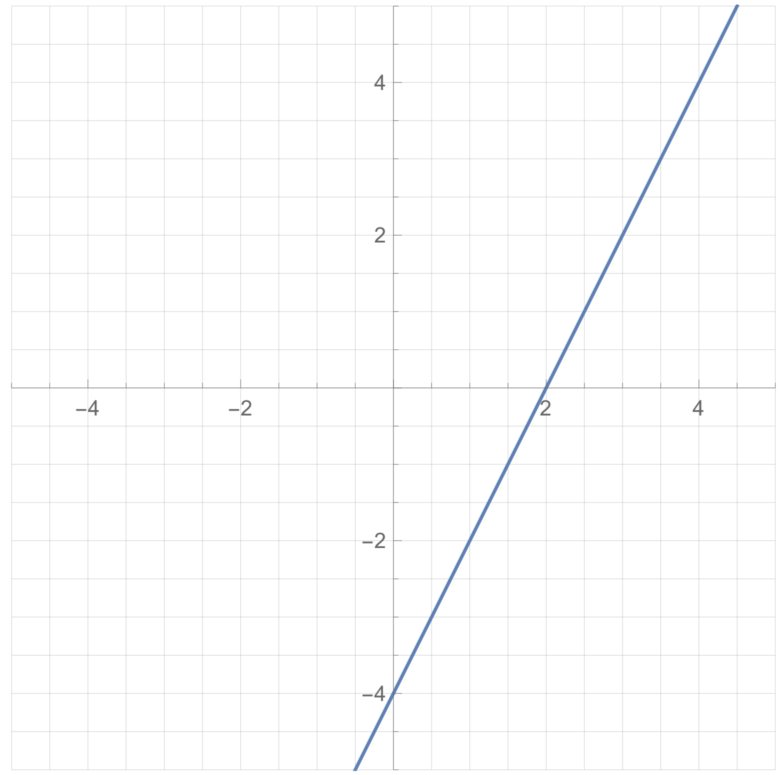
Example 3. *Graph the inequality*

$$6x - 3y \geq 12$$

Solution. *The line we want to graph is*

$$6x - 3y = 12 \quad \text{or} \quad y = 2x - 4.$$

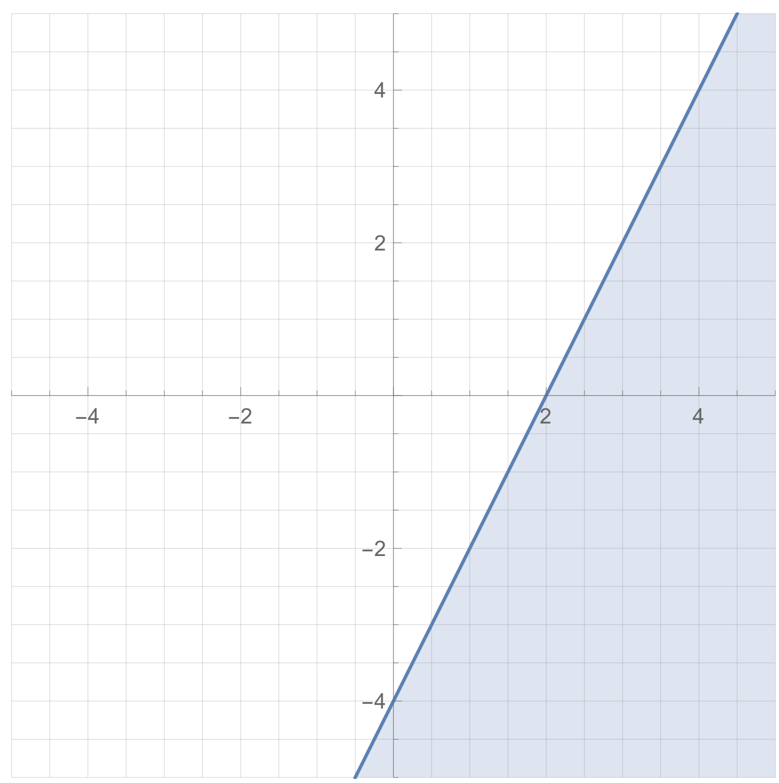
Since the inequality is not strict, we graph it with a solid line.



The point $(0,0)$ is not on the line, so we check that point in the inequality

$$6(0) - 3(0) = 0 \geq 12$$

This is false, so we shade in the side of the line without the origin.



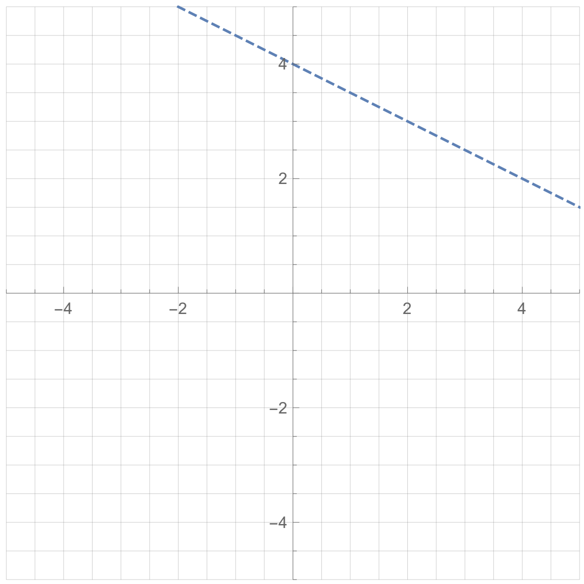
Example 4. *Graph the inequality*

$$4x + 8y < 32$$

Solution. *The line we want to graph is*

$$4x + 8y = 32 \quad or \quad y = -\frac{1}{2}x + 4.$$

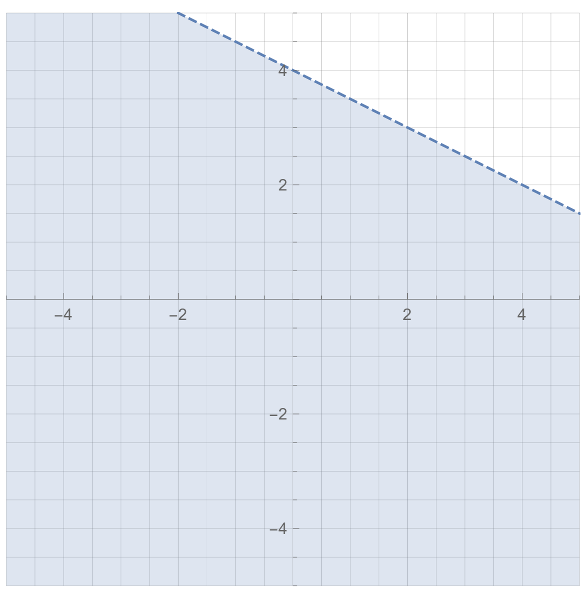
Since the inequality is strict, we graph it with a dashed line.



The point $(0, 0)$ is not on the line, so we check that point in the inequality

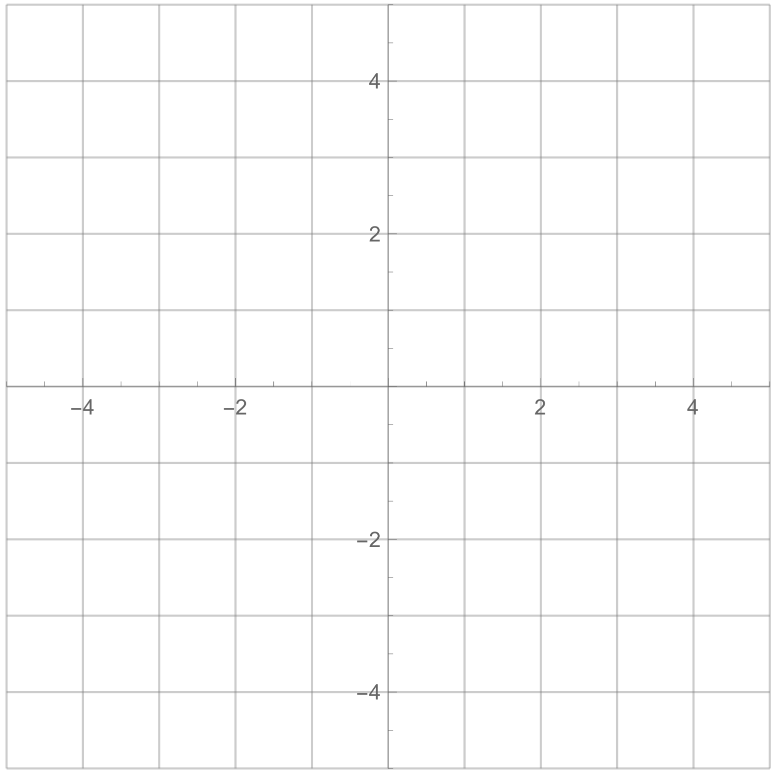
$$4(0) + 8(0) = 0 < 32$$

This is true, so we shade in the side of the line with the origin.



Example 5. *Graph the inequality*

$$2y \leq 10$$



Example 6. *Graph the inequality*

$$2x - 5y > 10$$